Mathematics - Course 121

THE BINOMIAL DISTRIBUTION AND POWER SYSTEM RELIABILITY

I THE BINOMIAL DISTRIBUTION

Recall (from 121.00-3) that the number of different combinations of r objects which can be formed from n different objects is

$$_nC_r = \frac{n!}{r!(n-r)!}$$

Suppose that an 'experiment' is tried repeatedly and that all of the following apply:

1. the total number of trials, n, is fixed.
2. each trial has only two possible outcomes:
   - 'success' with probability p, and
   - 'failure' with probability q,

   where $p + q = 1$.
3. p and q are the same for all trials.
4. the outcome of any one trial is independent of all others.

Then the probability of exactly r 'successes' is

$$P_r = {_nC_r}p^rq^{n-r}.$$

In this expression, p^rq^{n-r} represents the probability (given by PR1) of exactly r 'successes' and n-r 'failures' in some particular order, eg, r consecutive 'successes' followed by n-r consecutive 'failures'. The coefficient $_nC_r$ accounts for the number of possible ways to order r 'successes' and n-r 'failures'.

The distribution of probability over the n+1 possible numbers of successes, r = 0, 1, 2, ... n, is called the *Binomial Distribution*. The binomial distribution takes its name from the binomial expansion,

$$(p + q)^n = \sum_{r=0}^{n} {_nC_r}p^rq^{n-r}.$$

Example 1

Plot the probability distribution of the number of heads obtained in 5 tosses of a coin.

Solution

In this case, $n = 5$, and $p = q = 1/2$

$$P_0 = {}_5C_0 p^0 q^5 = \frac{1}{32}$$

$$P_1 = {}_5C_1 pq^4 = \frac{5}{32}$$

$$P_2 = {}_5C_2 p^2 q^3 = \frac{10}{32}$$

$$P_3 = {}_5C_3 p^3 q^2 = \frac{10}{32}$$

$$P_4 = {}_5C_4 p^4 q = \frac{5}{32}$$

$$P_5 = {}_5C_5 p^5 q^0 = \frac{1}{32}$$

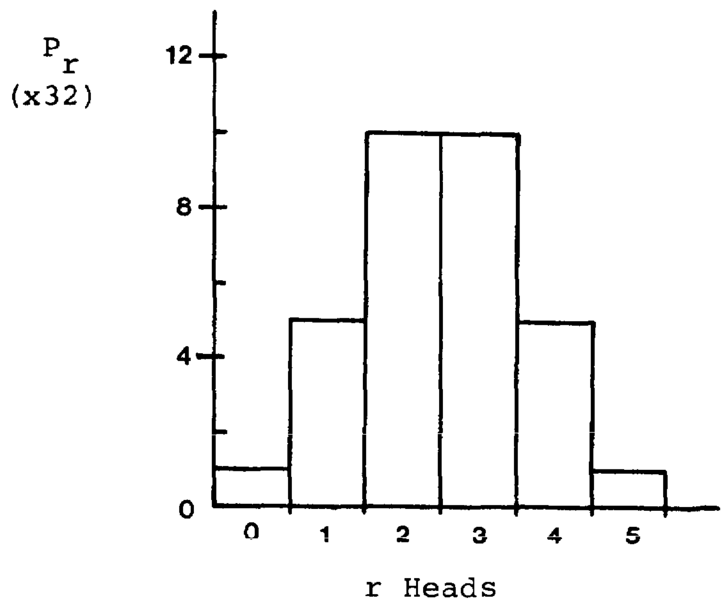


Figure 1

Probability Distribution of Number of Heads in 5 Coin Tosses

Example 2

Using Probability Rule #9 from 121.00-3, prove that the mean of the binomial distribution is np.

Proof

$$E(r) = \sum_{r=0}^{n} rP_r \qquad \text{(by PR9)}$$

$$= \sum_{r=1}^{n} r \frac{n!}{r!(n-r)!} p^r q^{n-r} \quad (\because \text{ no contribution from } r=0)$$

$$= np \sum_{r=1}^{n} \frac{(n-1)!\left(p^{r-1} q^{[(n-1)-(r-1)]}\right)}{(r-1)![(n-1)-(r-1)\ !]}$$

$$= np \sum_{s=0}^{m} \frac{m!}{s!(m-s)!} p^s q^{m-s} \qquad \begin{matrix}(m = n - 1)\\(s = r - 1)\end{matrix}$$

$$= np\ (p + q)^m \qquad \text{(using binomial expansion)}$$

$$= np(1)^m \qquad (p + q = 1)$$

$$= np \qquad \text{QED}$$

Example 3

Plot the probability distribution of successes in 4 trials, given that p = 1/4. What is the expected number of successes?

Solution

$$(p + q)^4 = {}_4C_0p^0q^4 + {}_4C_1pq^3 + {}_4C_2p^2q^2 + {}_4C_3p^3q + {}_4C_4p^4q^0$$

$$= \frac{81}{256} + \frac{108}{256} + \frac{54}{256} + \frac{12}{256} + \frac{1}{256}$$

$$P_0 \qquad P_1 \qquad P_2 \qquad P_3 \qquad P_4$$

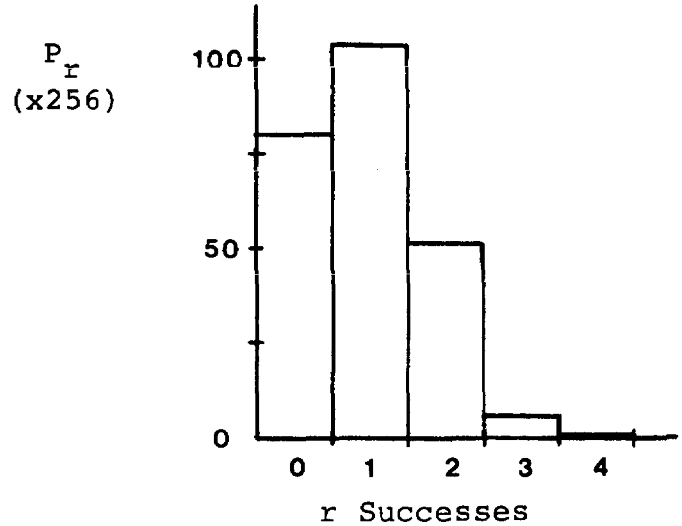


Figure 2

Probability Distribution of
Successes in 4 Trials

Using the result of Example 2 above, the expected number of successes, $E(r) = np$

$$= 4 \times 1/4$$
$$= 1$$

Example 4

If stock is 99.5% free of defects, what is the expected number of defective items in a random sample of 20 items?

Solution

Probability of defect in each item examined, $p = 0.005$.
Then expected number of defects $= np$

$$= 20 \times 0.005$$
$$= 0.1$$

Example 5

A system consisting of 8 dousing valves is considered failed if 3 or more dousing valves have failed. If the unavailability of a dousing valve is 0.01, calculate the unavailability of the system.

Solution

Let Q_v, Q_s represent unavailabilities of a dousing valve, dousing system, respectively. Then

Q_s = P(exactly 3 valves failed) + P(exactly 4 valves failed)

+ ... + P(exactly 8 valves failed) (by PR4)

$$= {}_8C_3 Q_v^{\ 3} R_v^{\ 5} + {}_8C_4 Q_v^{\ 4} R_v^{\ 4} + \ldots + {}_8C_8 Q_v^{\ 8}$$

$$= \frac{8!}{3!5!}(.01)^3(.99)^5 + \frac{8!}{4!4!}(.01)^4(.99)^4 + \ldots + \frac{8!}{8!0!}(.01)^8$$

$$= 5.3 \times 10^{-5} + 6.7 \times 10^{-7} + \ldots + 10^{-16}$$

$$= \underline{\underline{5 \times 10^{-5}}}$$

Note that only the lead term on the RHS contributed significantly to the answer, the probabilities of higher order failures (failure of 4, 5, ..., 8 valves) being negligible by comparison to the probability of 3 valves failing.

II POWER SYSTEM RELIABILITY

The average daily variation of grid load is shown schematically in Figure 3.

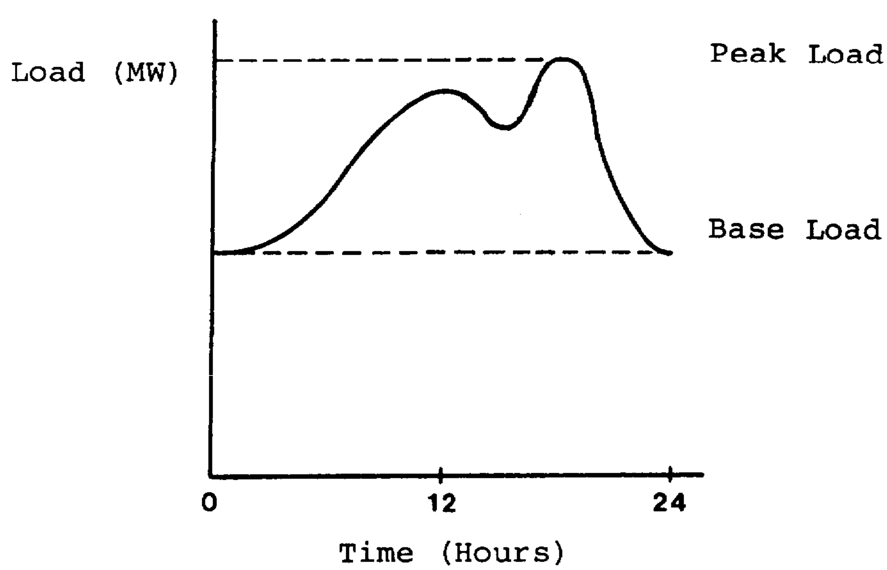


Figure 3

Average Daily Load Variation

A utility very seldom gets into trouble supplying its base load, but is much more likely to encounter difficulty supplying the peak load. (Hence, the move towards providing incentives for switching to off-peak consumption.) Figure 4 shows the number of days per annum that daily peak load exceeds any given value. Also shown on Figure 4 is the installed capacity, the reserve capacity (installed minus maximum daily peak), and the lost capacity, O_k, associated with the kth possible outage, and t_k, the number of days per annum that O_k causes a load loss.

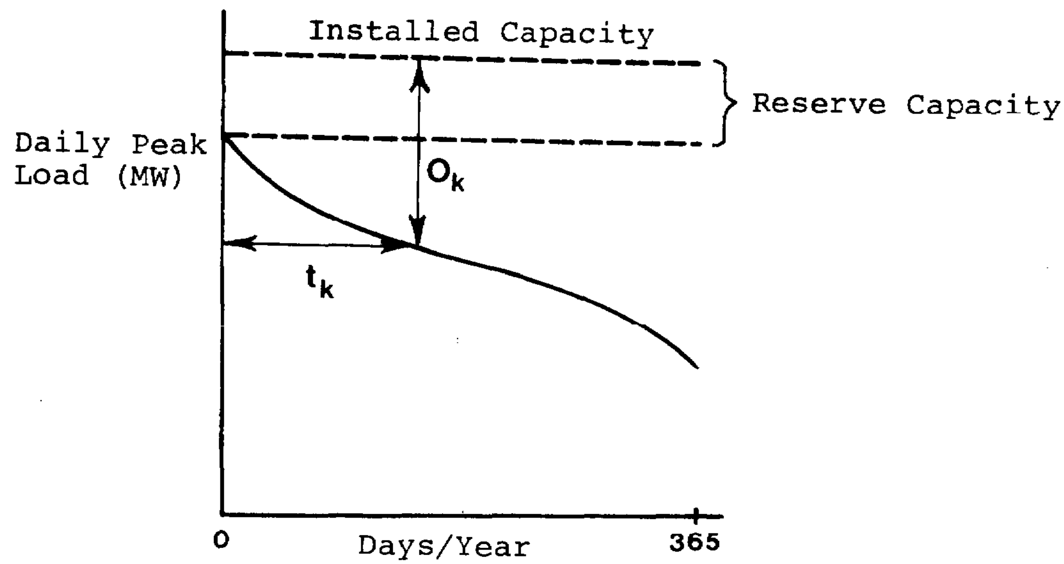


Figure 4

Typical Annual Daily Peak Load Variation Curve

DEFINITION

The *Expected Load Curtailment* (ELC) is the number of days per annum for which a load loss occurs, ie,

$$\boxed{ELC = \sum_k P_k t_k}$$

where P_k is the probability of the kth possible outage, and t_k is the number of days/year that the kth outage causes a load loss.

DEFINITION

The *Expected Load Loss* (ELL) is the expectation value of the load loss without regard to time, ie,

$$\boxed{ELL = \sum_k P_k L_k}$$

where P_k is the probability of the kth outage, and

L_k is the load lost on the kth outage.

Both ELC and ELL are used as measures of power system reliability. A utility would set a reliability target of, say, <0.1 day/y for the ELC.

DEFINITION

Forced outage rate is the fraction of time on forced outage, ie,

$$FOR = \frac{\text{Time on forced outage}}{\text{Time on forced outage + Time in operation}}$$

Note that for simplicity, this lesson assumes that generating units are either operating at full output or are down on forced outage. Other possible states such as 'operating derated' or 'available but not operating' are not considered.

Example 6

A small power system consists of three identical 5 MW generating units, each with a forced outage rate (FOR) of 0.03. Assuming a continuous load of 10 MW, draw up a capacity outage probability and expected load loss distribution table. Calculate the expected load loss and the expected load curtailment.

Solution

The Annual Daily Peak Load Variation Curve for this example is shown in Figure 5, and the required capacity outage probability and load loss distribution table is given in Table 1.

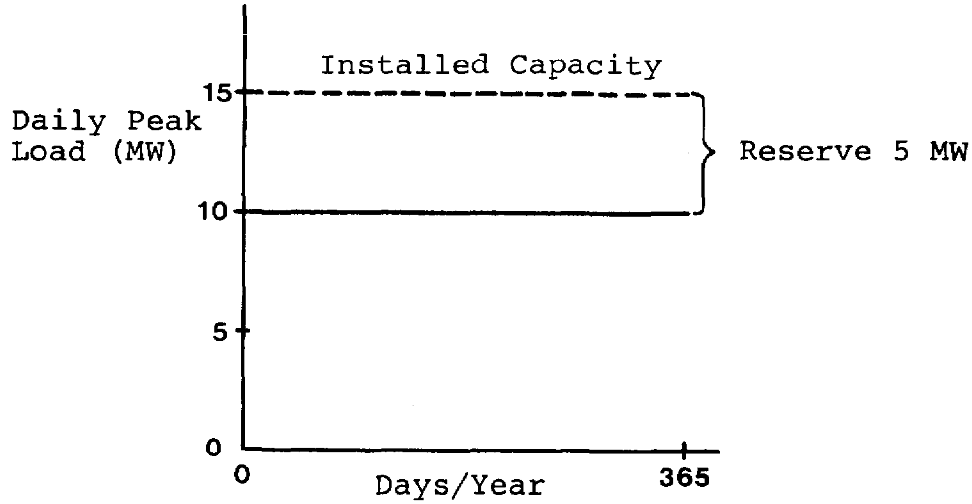


Figure 5

Annual Daily Peak Load
Variation Curve for Example 6

Note that any outage O_k resulting in a load loss L_k has an associated t_k = 365 days/y.

The ELL and ELC are calculated using Table 1 as follows:

$$ELL = \sum_k L_k P_k$$

$$= \underline{\underline{0.013365 \text{ MW}}}$$

$$ELC = \sum_{k}^{\text{all losses}} P_k t_k$$

$$= (0.002619 + 0.000027)\ (8760 \text{ hours/y})$$

$$= \underline{\underline{23 \text{ hours/y}}}$$

Example 6

Table 1

Capacity Outage Probability and Load Loss Distribution Table

k	Outage O_k		Probability P_k	Load Loss L_k (MW)	L_kP_k(MW)
	Number Units Out	Capacity Out (MW)			
1	0	0	0.912673	0	0
2	1	5	0.084681	0	0
3	2	10	0.002619	5	0.013095
4	3	15	0.000027	10	0.000270

Note $\sum_k P_k = \sum_r {}_3C_r(.97)^r(.03)^{3-r} = (.97 + .03)^3 = 1.000000$

That $\sum_k P_k = 1$ can be used as an internal check when drawing up such tables.

Example 7

A power system contains the following generating capacity:

3 x 40 MW hydro units, FOR = 0.005

1 x 50 MW thermal unit, FOR = 0.02

1 x 60 MW thermal unit, FOR = 0.02

a) Calculate the ELL and ELC assuming a continuous load of 200 MW.

b) Calculate the ELC assuming the annual daily peak load variation curve decreases linearly from 200 MW at t = 0 to 80 MW at t = 365 d/y.

Solution

a) The complete capacity outage probability distribution table including load losses for the case of a 200 MW continuous load is given in Table 2.

From Table 2,

$$ELL = \sum_k P_k L_k$$

$$= \underline{\underline{1.181975 \text{ MW}}}$$

$$ELC = \sum_k^{\text{all losses}} P_k t_k$$

$$= (\sum_{k=2}^{16} P_k)\ (365\ d/y) \quad (t_k = 365\ d/y,\ k = 2,\ldots,16)$$

$$= \underline{\underline{19.7 \text{ days/year}}}$$

(Note that $\sum_{k=2}^{16} P_k = 1 - P_1$ in the above.)

Example 7(a)

Outage O_k						
k	Units	Capacity	P_k		L_k	$P_k L_k$
1	none	0	$(.995)^3(.98)^2$	.946066	0	0
2	40	40	${}_3C_1(.995)^2(.005)(.98)^2 =$	.014262	10	.142623
3	50	50	$(.995)^3(.02)(.98) =$	.019307	20	.386148
4	60	60	$(.995)^3(.98)(.02) =$	.019307	30	.579224
5	2 x 40	80	${}_3C_2(.995)(.005)^2(.98)^2 =$	.000072	50	.003583
6	40, 50	90	${}_3C_1(.995)^2(.005)(.02)(.98) =$	.000291	60	.017464
7	40, 60	100	${}_3C_1(.995)^2(.005)(.98)(.02) =$	.000291	70	.020375
8	50, 60	110	$(.995)^3(.02)^2 =$	.000394	80	.031522
9	3 x 40	120	${}_3C_3(.005)^3(.98)^2 =$	.000000	90	.000011
10	2x40,50	130	${}_3C_2(.995)(.005)^2(.02)(.98) =$	.000001	100	.000146
11	2x40,60	140	${}_3C_2(.995)(.005)^2(.98)(.02) =$	.000001	110	.000161
12	40,50,60	150	${}_3C_1(.995)^2(.005)(.02)^2 =$	.000006	120	.000713
13	3x40,50	170	$(.005)^3(.02)(.98) =$	.000000	140	.000000
14	3x40,60	180	$(.005)^3(.98)(.02) =$	.000000	150	.000000
15	2x40,50,60	190	${}_3C_2(.995)(.005)^2(.02)^2 =$	.000000	160	.000005
16	3x40,50,60	200	$(.005)^3(.02)^2 =$	.000000	200	.000000

Table 2

Capacity Outage Probability Distribution and Load Loss Table

b) The annual daily peak load variation curve is shown in Figure 6.

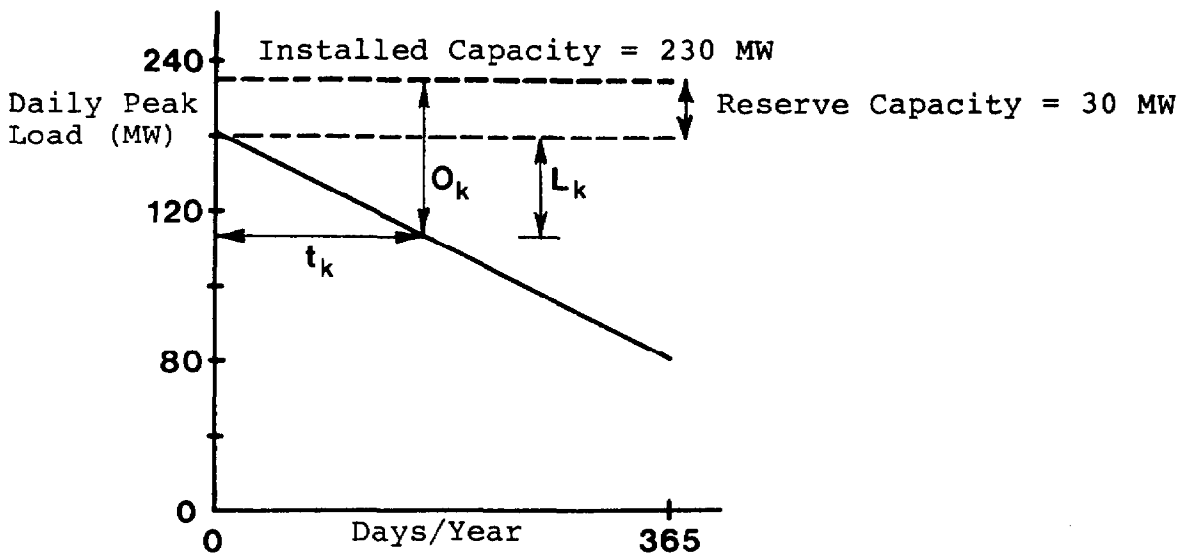


Figure 6

Annual Daily Peak Load Variation Curve for Example 7(b)

By similar triangles in Figure 6,

$$\frac{L_k}{t_k} = \frac{120\ \text{MW}}{365\ \text{d/y}}$$

ie, $$t_k = \frac{365}{120} L_k$$

The complete capacity probability and load loss table for Example 7(b) is given in Table 3, from which

$$\text{ELC} = \sum_{k=2}^{16} P_k t_k$$

$$= \underline{\underline{3.6 \text{ days/year}}}$$

Example 7 shows that, even with a few units, the capacity outage probability distribution and load loss table can become complex. Examination of Tables 2 and 3 shows, however, that there are only a few outages which contribute significantly to the expected load curtailment. In practice, the table is truncated at some specified cumulative probability, say, for example, at 99.9999%. This truncation might cut out hundreds or even thousands of entries in the table for a large power system.

Example 7(b)

k	O_k Units	O_k Capacity	P_k	L_k	$t_k = \frac{365}{120} L_k$	$P_k t_k$
1	0	0	.946066	0	0	0
2	1 x 40	40	.014262	10	30.4167	.433803
3	50	50	.019307	20	60.8333	1.174509
4	60	60	.019307	30	91.2500	1.761764
5	2 x 40	80	.000072	50	152.0833	.010950
6	40, 50	90	.000291	60	182.5000	.053108
7	40, 60	100	.000291	70	212.9167	.061959
8	50, 60	110	.000394	80	243.3333	.095873
9	3 x 40	120	.000000	90	273.7500	.000033
10	2x40,50	130	.000001	100	304.1667	.000304
11	2x40,60	140	.000001	110	334.5833	.000335
12	40,50,60	150	.000006	120	365	.002190
13	3x40,50	170	.000000	140	365	.000001
14	3x40,60	180	.000000	150	365	.000001
15	2x40,50,60	190	.000000	160	365	.000004
16	3x40,50,60	230	.000000	200	365	.000000

Table 3

Capacity Outage Probability Distribution and Load Loss Table

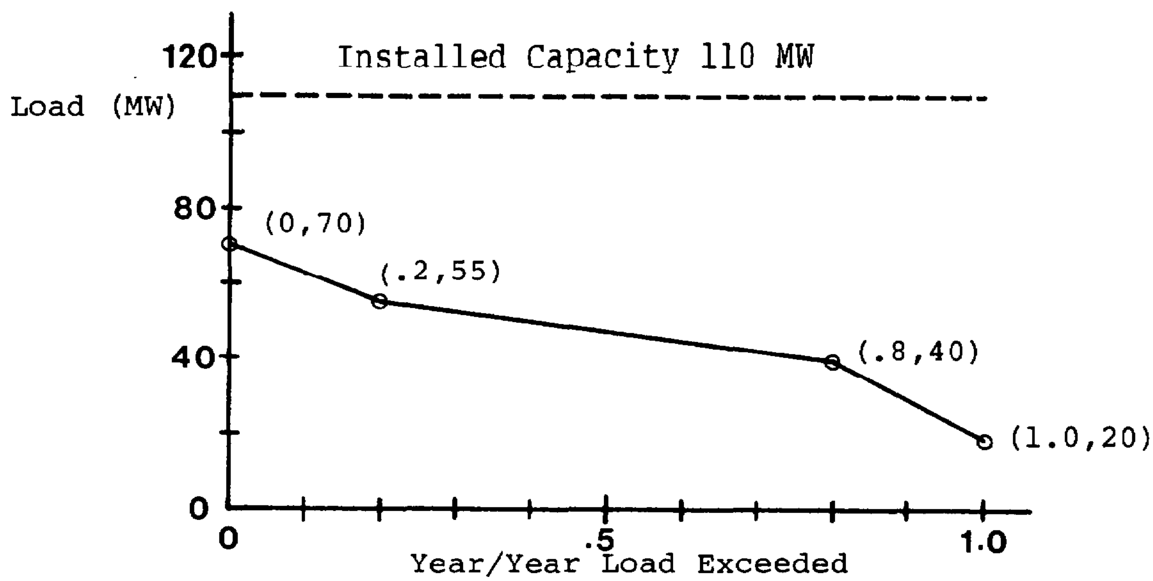
ASSIGNMENTS

1. A dousing system consisting of 14 identical dousing valves is considered failed if 3 or more valves are failed. If valve availability is 0.99, calculate the system unavailability, and compare with that of Example 5 in the text.

2. A power system has two generating units:

   - A with capacity 50 MW and availability 0.95, and
   - B with capacity 60 MW and availability 0.96.

   The annual load variation curve for the system is shown below:



   a) Draw up a capacity outage probability distribution table and calculate the expected load curtailment for the system.

   b) A 10 MW generator C with availability 0.96 is added to the system. Recalculate the ELC and explain why such a relatively small change in overall capacity effects such a large reduction in the ELC.

3. For each of the following transformer systems, calculate and compare

   a) The expected percentage load curtailment.

   b) The expected load curtailment in hours/year. Assume all transformers have a FOR of 0.015.

   i) 3 transformers each rated at 100% of full load.

   ii) 3 transformers each rated at 90% of full load.

   iii) 3 transformers each rated at 50% of full load.

   iv) 4 transformers each rated at 33 1/3% of full load.

L. Haacke